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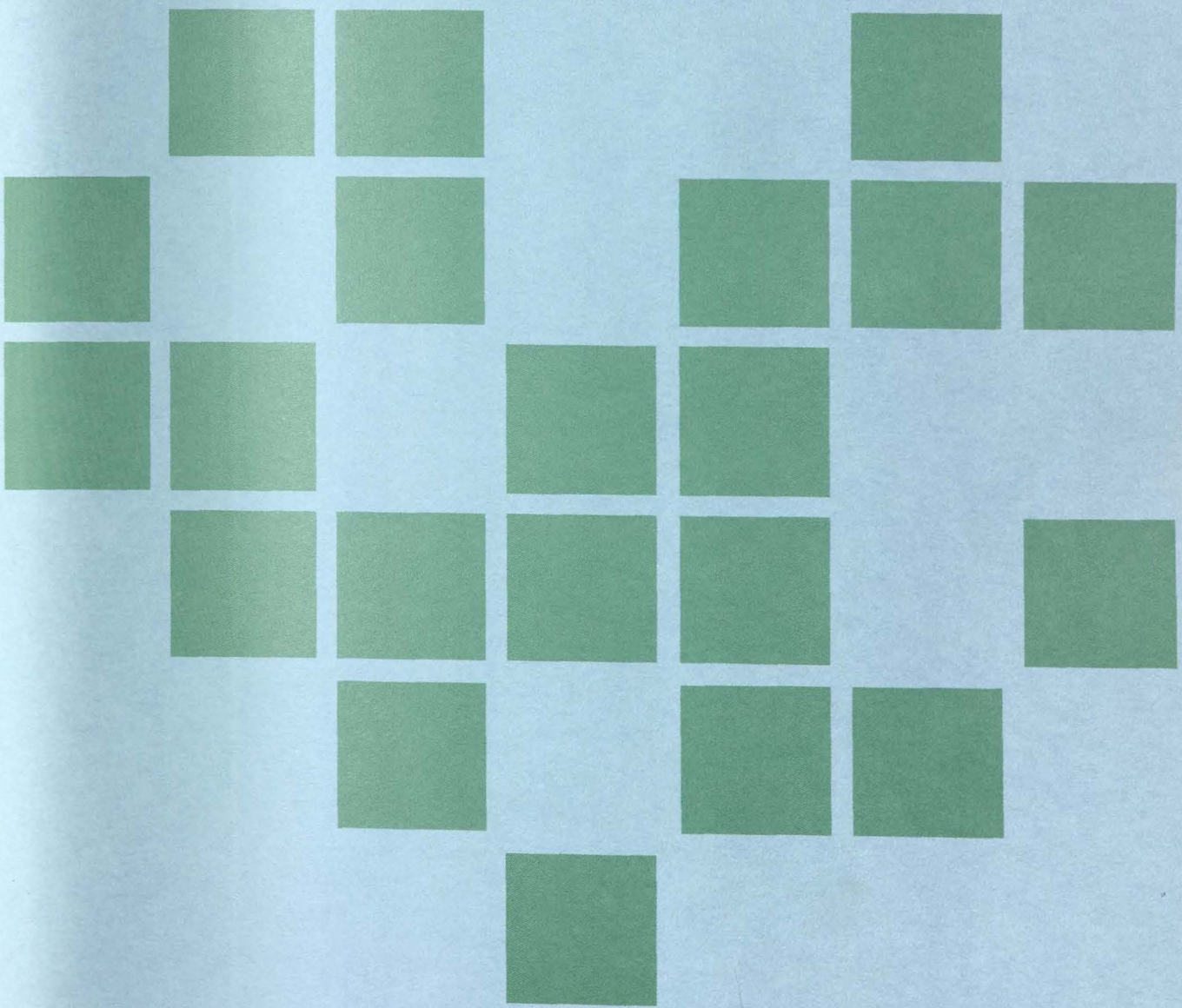
Stephen R. Shifley, John W. Moser, Jr., and Kenneth M. Brown



North Central Forest Experiment Station
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GROWTH AND YIELD MODEL FOR THE ELM-ASH-COTTONWOOD TYPE IN INDIANA

Stephen R. Shifley, *Research Forester,*
North Central Forest Experiment Station, St. Paul, Minnesota,
John W. Moser, Jr., *Professor of Forestry,*
Purdue University, West Lafayette, Indiana,
and **Kenneth M. Brown,** *Associate Professor of Forestry,*
Lakehead University, Thunder Bay, Ontario

More than 0.5 million acres of commercial elm-ash-cottonwood forest can be found in Indiana (Spencer 1969) with an additional 1.4 million acres occurring in Illinois (Essex and Gansner 1965). The percent of commercial elm-ash-cottonwood forest land in these two States is surpassed only by that of oak-hickory. Most elm-ash-cottonwood stands occur on good to excellent sites. Despite its abundance, however, little has been published concerning the growth and yield of the elm-ash-cottonwood (E-A-C) forest type in the Central States (Shifley and Brown 1978).

This paper reports the results of a recent study of the growth and yield of the E-A-C forest type in Indiana. The study led to the development of a system of mathematical models that have been used to generate yield tables for E-A-C stands.

TYPE DEFINITION

The E-A-C forest type is defined here as any bottomland forest in which elm (*Ulmus* spp.), green ash (*Fraxinus pennsylvanica* Marsh.), cottonwood (*Populus deltoides* Marsh.), silver maple (*Acer saccharinum* L.), or red maple (*Acer rubrum* L.), singly or in any combination, comprise a plurality of the stocking. Willow (*Salix nigra* Marsh.), boxelder (*Acer negundo* L.), and sycamore (*Platanus occidentalis* L.) are common associates. Although the E-A-C species mixture inhabits a variety of poorly drained upland sites, only occurrences of the type adjacent to waterways were considered in this study.

DATA

Remeasurement data were collected in 1977 from 35 continuous forest inventory (CFI) sampling units located in Indiana E-A-C stands of natural origin.[1] Each sampling unit was comprised of 10 subsamples equally spaced over approximately 1 circular acre. At each subsample point trees greater than or equal to 5 inches d.b.h. were sampled using a 37.5 factor prism. Trees between 1 and 5 inches d.b.h. were sampled on a 1/300 acre circular plot centered on the prism point. Annual ingrowth, survivor growth, mortality, and cut were estimated on a per-acre basis for each of the sampling units (table 1). These annual rates of change were used to construct the rate equations in subsequent sections.

Although pure stands of even-aged cottonwood, willow, or mixtures of these two species are classic early stages in E-A-C succession on newly-formed land, the stands encountered in the data collected for this study are characterized by mixed species and variable size classes as found in subsequent seral stages or in E-A-C development on old field sites (Hosner and Minkler 1963).

PAST WORK

Whole-stand, age-independent modeling techniques are well suited to predict E-A-C stand dynamics because such models can be calibrated with few data and for both even-aged and uneven-aged stands.

MacKinney *et al.* (1937) and Schumacher (1939) present some of the earliest mathematical forest stand growth and yield models. It was not until more recently, however, that Buckman (1962) and Clutter

[1]*Data from Indiana State Survey were provided by the Renewable Resources Evaluation Project, North Central Forest Experiment Station.*

(1963) demonstrated that differential equations could be applied to obtain compatible growth and yield estimates for even-aged forest stands. Pienaar (1965) and Turnbull (1963) examine in detail the biomathematical aspects of growth and yield estimation in pure and mixed stands. Pienaar and Turnbull (1973) emphasize the importance of biologically reasonable growth and yield models, and they suggest the Chapman-Richards function as a suitable model for a variety of even-aged stand growth applications. While previous attempts to model stand growth using differential equations had been limited to even-aged stands, Moser and Hall (1969) extended differential equation theory to applications involving growth and yield in uneven-aged stands.

Leary (1970) suggests that systems for modeling forest stand dynamics may be separated into components, each component defined by one or more difference equations within a system of equations which, in total, describe stand growth. In such a system stand yield is determined through the simultaneous solution of the component equations. Leary gives an example in which tree size classes are the components through which stand dynamics are modeled, but he suggests that the components used to model stand dynamics will vary with the application. Moser (1972) presents a system of differential equations describing uneven-aged stand growth in which the components used are ingrowth, survivor growth, and mortality in the stand, i.e., the components of growth (Beers 1962).

Both Moser (1974) and Ek (1974) have demonstrated that more complex stand modeling systems are feasible. In separate studies involving applications with uneven-aged mixed hardwoods, each author models stand growth by dividing the stand into a number of component size classes. Growth within each size class is further separated into expressions for ingrowth, survival growth, mortality, and upgrowth. Moser uses a system of 66 equations (11 equations for each of 6 size classes) to directly simulate change in number of trees, basal area, and cubic foot volume. Ek achieves a similar goal with fewer equations by simulating change in the number of trees by 2-inch diameter classes and then estimating basal area and volume from the known number of trees in each diameter class.

Markov models offer an alternative to stand growth simulation by differential equations (Peden *et al.* 1973, Bruner and Moser 1973). Although Markov models characterizing stand dynamics may serve many useful purposes, they generally do not provide the same opportunities for theorization that are offered by differential equation-based models.

THE ELM-ASH-COTTONWOOD MODEL

A system of differential[2] equations was sought that would simulate change in the number of trees and basal area of E-A-C stands. Ideally, such a model should be the simplest one that describes the biological phenomena (Milsum 1966) and remains consistent with the structure and function of the actual biological system (Pienaar 1965). The methods used here are similar to those of Moser (1972) that divide stand growth into the components of ingrowth, mortality, and survivor growth. Modeling stand growth by diameter classes was considered infeasible due to the small amount of data available to calibrate the model.

The following notation will be used to describe the model.

dN_i/dt = Rate at which ingrowth trees cross the 5-inch d.b.h. threshold (units: trees/acre/year).

dN_m/dt = Rate at which trees 5 inches d.b.h. or larger die (units: trees/acre/year).

N = Total number of trees 5 inches d.b.h. or larger (units: trees/acre).

dB_i/dt = Basal area represented by the ingrowth trees (units: square feet/acre/year).

dB_m/dt = Basal area represented by the mortality trees (units: square feet/acre/year).

dB_s/dt = Basal area growth on the surviving N trees (units: square feet/acre/year).

B = Total basal area of the N trees (units: square feet/acre).

The net rate of change in the number of trees per acre per year can be analyzed as the algebraic sum of the ingrowth and mortality rates (Moser 1972):

$$\frac{dN}{dt} = \frac{dN_i}{dt} - \frac{dN_m}{dt} \qquad (1)$$

The net rate of change in basal area per acre per year can be analyzed similarly. Its components are the rates of change due to ingrowth, mortality, and survivor growth:

$$\frac{dB}{dt} = \frac{dB_i}{dt} - \frac{dB_m}{dt} + \frac{dB_s}{dt} \qquad (2)$$

[2]*Although a difference equation formulation is theoretically appropriate, we favored differential equations for analytical convenience. Numerical differences in solutions due to this choice are small.*

Table 1.—*Initial distribution of calibration data by basal area and number of trees per acre (trees 5 inches d.b.h. and larger)*

Basal area (ft²/acre)	Trees (No./acre)											
	20	40	60	80	100	120	140	160	180	200	220	240
10	2											
20		1	1									
30					1							
40			1	1		1						
50		1			1		1					
60					4	1	1	1				
70						1		2				
80				1		2	1		3			
90						2					1	
100									1			
110									1			
120												
130												1
140								1				

Equations (1) and (2) form the basis for a system of interactive differential equations that describe forest stand dynamics. To be of practical value, however, the derivatives on the right-hand sides of Equations (1) and (2) must be replaced by differential equations that are both biologically plausible and mathematically tractable. The development of these equations is presented in the following sections.

Ingrowth Rate: Number of Trees

Ingrowth occurs when trees enter the smallest measured diameter class from below. In this case a tree is classified as an ingrowth tree when it reaches 5 inches d.b.h. In real forest stands, ingrowth is a discrete phenomenon—an all or none event. However, for the stand model being developed here, ingrowth is treated as if it were a continuous process. Over the long run, the model predicts reasonable numbers of ingrowth trees.

Studies have indicated that stand density is a major factor influencing the number of seedlings and saplings in a stand. Minckler (1958) reports an increase in the number of saplings (0.6 to 4.6 inches d.b.h.) after E-A-C stand basal area was reduced by 32, 63, and 100 percent. Belanger and Pepper (1978) report a significant decrease in diameter growth of sycamore seedlings as number of trees per acre increase.

Because E-A-C stands are characteristically comprised of species that are intermediately tolerant or intolerant of competition, it is hypothesized that the rate of ingrowth can be characterized as a function of stand density. Both Ek (1974) and Moser (1972) have used negative exponential functions to express the periodic rate of ingrowth as a function of stand density. Examination of our E-A-C data showed that the annual rate of ingrowth decreased as basal area increased and that the rate of ingrowth was higher in stands with a small average diameter. These relations can be expressed as:

$$\frac{dN_i}{dt} = 10.14e^{(0.002456N - 0.01560B)} \qquad (3)$$

MSE (Mean Square Error) = 14.6

e = base of natural logarithms.

The greatest annual rates of ingrowth are predicted to occur in stands with large numbers of small diameter trees. According to Equation (3), if all of the trees in a stand are less than 5 inches d.b.h., (N and B both equal 0), the ingrowth rate would be about 10 trees per acre per year. Actual forest stands containing no trees more than 5 inches d.b.h. can range from bare ground to stands with thousands of saplings per acre. Such widely different stand conditions would produce different numbers of ingrowth trees. Therefore, Equation (3) should not be used to estimate ingrowth in stands where most of the trees are less than 5 inches d.b.h.

Mortality Rate: Number of Trees

Numerous authors have attempted to predict individual tree mortality, and they have found that the mortality rate decreases as tree size increases, growth rate increases, vigor improves, or competition decreases (Monserud 1976, Hamilton and Edwards 1976, Buchman 1979). In stand simulation models, the number of mortality trees has been expressed variously in functions that predict increases in mortality with increases in number of trees and/or basal area (Ek 1974, Moser 1974). Lee (1971) related mortality in even-aged stands to stand age and mean stand diameter.

The annual number of E-A-C mortality trees was positively correlated with density as measured by the number of trees and the basal area per acre. Annual mortality was expressed as separate functions of basal area and number of trees but several models combining these two independent variables

into one function did not improve statistical accuracy or biological reasonableness. Consequently, the following model was judged a suitable expression for the annual number of E-A-C mortality trees per acre.

$$\frac{dN_m}{dt} = 1.106e^{0.006561B} \qquad (4)$$

MSE = 2.7

Ingrowth Rate: Basal Area

The rate of basal area increase due to ingrowth is a direct function of the rate at which ingrowth trees occur. A 5-inch d.b.h. tree contributes 0.1364 square foot of basal area, so the relation between dN_i/dt and dB_i/dt is given by the differential equation:

$$\frac{dB_i}{dt} = 0.1364 \frac{dN_i}{dt} \qquad (5)$$

Because ingrowth is treated as a continuous process in the solution of this system of equations, that portion of the remeasured ingrowth trees' basal area in excess of 0.1364 square foot per tree was treated as survivor growth when the E-A-C data were analyzed.

Mortality: Basal Area

Mortality trees occur in all diameter classes, so a single large mortality tree may account for the same basal area as several smaller ones. Moser (1972) estimated the basal area of mortality trees by specifying a diameter distribution and assigning an appropriate diameter to each projected mortality tree.

Bailey and Dell (1973) used the Weibull probability density function to describe diameter distributions similar to those observed for E-A-C mortality trees. When properly calibrated, the cumulative form of the Weibull frequency distribution:

$$x = a + b[-\ln(1-F(x))]^{c-1} \qquad (7)$$

where

x = the tree diameter
a = location parameter
b = scale parameter
c = shape parameter

can be used to generate diameters of mortality trees if F(x) is replaced by a uniformly distributed random variable. The maximum likelihood parameter estimates of the distribution of mortality trees by diameter class, obtained by the method of Harter and Moore (1965), were a = 5.0, b = 3.142, and c = 0.9057. A Komolgorov-Smirnov test supported the null hypothesis that the fitted Weibull distribution is equivalent to the observed distribution at the α = 0.1 significance level. Consequently, the function

$$\frac{dB_m}{dt} = [5.0 + 3.142(-\ln(1-R))^{1.104}]^2 \; 0.005454 \; \frac{dN_m}{dt} \qquad (8)$$

where

R = a uniformly distributed random variable

generates a new basal area mortality estimate for each iteration in the solution of the system of equations.

This method of determining mortality basal area cannot be expected to perform well for stands that are very different from those from which the diameter distribution of mortality trees was obtained. The mean diameter for the remeasured stands was estimated to be between 8.5 and 14.2 inches at the 0.9 probability level. For the instances in which a simulated stand has a mean diameter of less than 8.5 inches or more than 14.2 inches, the mortality trees are assumed to be of average size for that stand. This avoids predicting unreasonably large or small mortality tree diameters.

Survivor Growth: Basal Area

The amount of basal area survivor growth is expected to depend upon the size and number of the survivor trees. Moser (1972) described stand basal area survivor growth using the Chapman-Richards function with stand basal area as the independent variable. Although stand basal area was correlated with the growth rate of E-A-C survivor trees, the best results were obtained by relating the basal area survivor growth to the stand sum of diameters. The sum of diameters for the stand was approximated as the product of the number of trees in the stand and the quadratic mean stand diameter, $N(B/(N \cdot .005454))^{1/2}$. Linear regression through the origin was used to obtain the expression for annual basal area growth of survivor trees,

$$\frac{dB_s}{dt} = 0.03043 (B \cdot N)^{1/2} \qquad (6)$$

$R^2 = .82^{**}$

MSE = 1.7

Although this function states that survivor basal area growth increases proportionately for all increases in the sum of diameters, simulated stand

growth does have an upper limit. As the basal area increases, stand mortality also increases to limit basal area growth.

RESULTS AND DISCUSSION

The entire system of differential equations describing the rates of change in number of trees and basal area for elm-ash-cottonwood stands is as follows:

$$\frac{dN_i}{dt} = 10.14e^{(0.002456N - 0.01560B)}$$

$$\frac{dN_m}{dt} = 1.106e^{0.006561B}$$

$$\frac{dN}{dt} = \frac{dN_i}{dt} - \frac{dN_m}{dt}$$

$$\frac{dB_i}{dt} = 0.1364\,\frac{dN_i}{dt}$$

$$\frac{dB_s}{dt} = 0.03043\,(B \cdot N)^{1/2}$$

$$\frac{dB_m}{dt} = [5.0 + 3.142(-1n(1-R))^{1.104}]^2$$

$$0.005454\,\frac{dN_m}{dt}$$

$$\text{if } 0.40 < \frac{B}{N} < 1.10$$

$$= \frac{B}{N}\,\frac{dN_m}{dt} \text{ otherwise}$$

$$\frac{dB}{dt} = \frac{dB_i}{dt} - \frac{dB_m}{dt} + \frac{dB_s}{dt}$$

The above system of equations was utilized with a fourth order Runge-Kutta algorithm (International Business Machines 1968) to develop a simulator that generates E-A-C growth and yield estimates in basal area and number of trees per acre. Inputs to the simulator are the initial values for basal area and number of trees. Outputs are the projected values for basal area and number of trees by growth components at user-specified time intervals. Tables 4 through 15 in Appendix I contain 25-year projections of total basal area, total number of trees, and cubic foot volume excluding bark for trees 5 inches d.b.h. and larger. Volume estimates were derived from the projected stand basal area using the following formula:

$$V = 8.319B^{1.211} \qquad (9)$$

where

V = cubic foot volume excluding bark for trees ≥ 5 inches d.b.h. and
B = basal area.

This function was fitted from the initial measured basal area and cubic foot volume in the E-A-C data. Tree volumes were estimated from the composite hardwood volume table formulae prepared by Beers (1964).

The yield tables in Appendix I are based on a single simulation trial. Because the basal area mortality function Equation (7) is stochastic, it will permit some variation in the final predictions. However, the simulation that produced the Appendix tables was performed in a manner such that from 50 to several hundred iterations were made during the solution process. A different-sized mortality tree was selected for each iteration and these small, numerous iterations made the effects of using a particular sequence of random numbers to specify mortality tree diameters almost negligible. Repeated trials with different sequences of random numbers produced results that varied by only 1 or 2 square feet of basal area after 25 years of simulation. However, it remains possible to alter the simulation program so that basal area mortality produces greater random variation in the basal area mortality during simulation trials. The FORTRAN program used for simulation is listed in Appendix II.

No data were reserved for independent validation and testing. Tests were conducted, however, to determine how well the model could predict the observed changes in stand characteristics for the 11-year remeasurement interval of the calibration data (table 2, fig. 1). Minckler's (1958) bottomland hardwood cutting study produced a small number of CFI plots suitable for independently validating the E-A-C model.[3] The study included three control plots, each approximately 2 acres in size. These plots were established in 1953 and last remeasured in 1968. The simulator was used to project the initial conditions for each of these plots to an estimate of their 1968 condition (table 3). Individual differences in the observed and predicted basal areas for the three plots after 15 years were +2, +12, and-19 ft²/acre. The greatest error occurred on a plot within an even-aged cottonwood stand.

A desirable property of any growth simulation model is that it gives reasonable results for long projection periods. There are no data to evaluate the

[3]*Data provided by Richard Schlesinger, North Central Forest Experiment Station, Carbondale, Illinois.*

Table 2.—*Eleven-year mean observed and mean predicted values for 35 elm-ash-cottonwood calibration plots (trees 5 inches d.b.h. and larger)*

	Trees	Basal area	Volume
	No./acre	*Ft²/acre*	*Ft³/acre*
Initial	109	62	[1]1,270
Observed	152	90	[1]2,053
Predicted	137	92	[2]2,017
Difference	-15	+2	-36
Std. error of difference	9	3.4	101

[1]Computed from the sum of observed individual tree volumes.
[2]Computed as a function of basal area, Equation (9).

accuracy of long-term E-A-C projections, but one may be encouraged if estimated stand conditions remain believable for a long projection period. Projections of two E-A-C stands for 150 years produced reasonable volume estimates (fig. 2). Although long-range projections provide an enlightening test of a model's capabilities, it is seldom prudent to rely on projections that are much longer than the interval represented by calibration data.

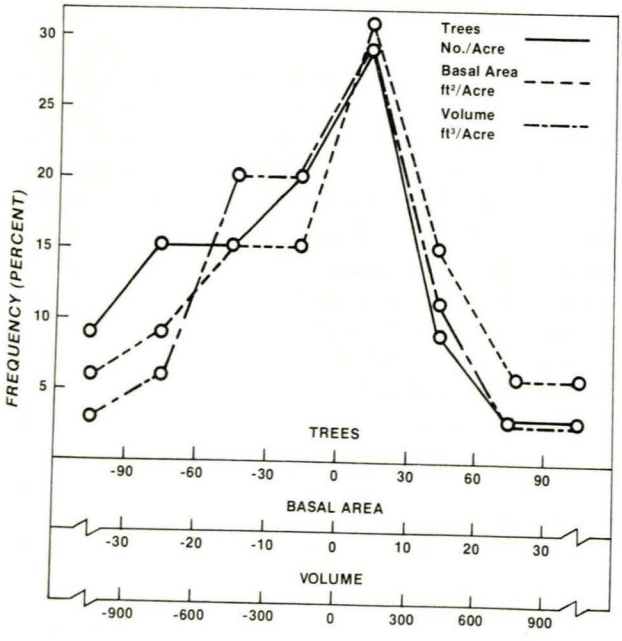

Figure 1.—*Frequency of errors in 11-year projections of 35 elm-ash-cottonwood calibration plots.*

Table 3.—*Fifteen-year mean observed and predicted values of trees 5 inches d.b.h. and larger for three elm-ash-cottonwood validation plots*

Item	Trees	Basal area	Volume
	No./acre	*ft²acre*	*ft³/acre*[1]
Initial	89	89	1,927
Final observed	83	114	2,600
Final predicted	96	113	2,533
Difference	+13	-1	-67

[1]Volumes computed as a function of basal area, Equation (9).

The growth models and simulated yield tables presented here outline some general patterns of E-A-C stand development in Indiana's bottomland. Additional research is necessary to document the effects of species composition, age structure, and management activities not addressed in this study but which certainly affect E-A-C stand growth. We hope that the information presented here will facilitate greater understanding and better utilization of the elm-ash-cottonwood forest resource.

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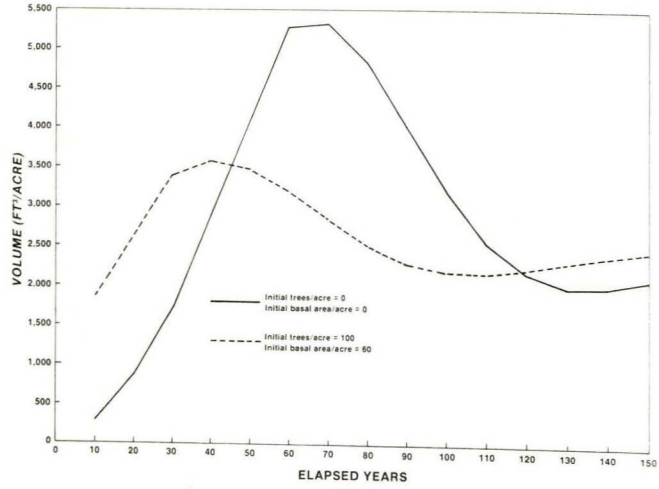

Figure 2.—*Projected volume of trees 5 inches d.b.h. and larger on two elm-ash-cottonwood plots.*

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APPENDIX I

SIMULATED ELM-ASH-COTTONWOOD YIELDS

Table 4.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 20 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
10	135	[1]59	96	128	155	175
		18	28	41	55	72
		268	471	740	1,074	1,466
20	313	52	82	109	130	147
		27	36	47	60	74
		446	637	878	1,177	1,532
30	512	47	71	93	110	123
		35	44	54	66	79
		619	810	1,044	1,329	1,658

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 5.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 40 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
10	135	[1]81	118	150	175	193
		19	31	46	62	79
		303	541	850	1,226	1,661
20	313	74	104	130	150	165
		28	39	52	66	82
		479	709	997	1,335	1,730
30	512	67	91	111	127	138
		38	48	60	73	88
		685	911	1,182	1,505	1,874
40	725	62	80	96	107	114
		48	58	68	81	94
		903	1,127	1,389	1,695	2,034
50	950	58	73	84	93	98
		56	65	75	87	99
		1,089	1,306	1,563	1,852	2,165
60	1,185	54	67	77	84	89
		61	69	79	90	98
		1,199	1,404	1,649	1,928	2,138

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 6.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 60 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
10	135	[1]103	141	172	196	212
		21	34	50	68	87
		330	602	950	1,370	1,849
20	313	95	126	151	170	182
		30	43	57	73	90
		518	787	1,118	1,507	1,948
30	512	89	113	133	148	157
		40	51	64	79	95
		716	976	1,286	1,651	2,065
40	725	83	102	117	127	133
		49	61	73	87	101
		938	1,201	1,508	1,859	2,240
50	950	78	92	102	109	112
		60	71	82	95	108
		1,717	1,442	1,738	2,061	2,421
60	1,185	73	84	91	94	95
		69	80	91	103	108
		1,415	1,674	1,960	2,267	2,412
70	1,428	70	78	83	88	91
		77	86	92	96	101
		1,592	1,827	1,985	2,107	2,225
80	1,679	68	75	81	86	89
		81	84	89	94	99
		1,713	1,787	1,909	2,047	2,164

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 7.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 80 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
20	313	[1]117 32 551	148 46 858	173 62 1,231	190 79 1,664	201 98 2,147
30	512	110 41 753	135 54 1,052	154 69 1,407	168 85 1,813	176 103 2,268
40	725	104 51 976	123 64 1,275	138 78 1,623	147 93 2,012	152 109 2,448
50	950	99 61 1,213	113 73 1,515	123 87 1,858	129 101 2,233	130 116 2,640
60	1,185	94 71 1,455	104 83 1,760	111 96 2,102	113 110 2,464	112 124 2,845
70	1,428	90 81 1,706	97 93 2,010	100 105 2,338	100 113 2,560	99 117 2,674
80	1,679	87 91 1,965	90 100 2,198	92 104 2,316	93 108 2,426	93 112 2,523
90	1,937	85 94 2,041	88 98 2,153	91 102 2,267	92 107 2,376	93 110 2,475
100	2,200	82 102 2,257	84 104 2,316	85 106 2,374	85 108 2,426	85 110 2,472

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 8.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 100 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
20	313	[1]138 33 580	171 49 923	195 66 1,338	212 85 1,816	221 106 2,350
30	512	131 43 790	157 58 1,130	176 74 1,527	188 92 1,982	194 111 2,485
40	725	125 52 1,003	145 66 1,337	159 82 1,726	168 99 2,166	171 116 2,643
50	950	120 63 1,246	135 76 1,586	144 91 1,967	149 107 2,385	150 123 2,833
60	1,185	115 72 1,489	126 86 1,830	132 100 2,208	134 115 2,615	132 131 3,048
70	1,428	111 83 1,745	118 96 2,093	120 110 2,473	119 125 2,870	115 133 3,116
80	1,679	107 93 2,007	111 106 2,355	111 120 2,735	108 126 2,905	104 129 3,005
90	1,937	104 103 2,275	105 115 2,618	103 120 2,747	101 124 2,846	97 126 2,910
100	2,200	101 112 2,523	100 117 2,649	99 120 2,748	96 123 2,819	93 124 2,859
110	2,469	100 115 2,597	98 118 2,701	96 121 2,778	93 123 2,826	90 124 2,846
120	2,744	97 123 2,824	94 125 2,873	91 125 2,890	87 125 2,878	84 124 2,842

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 9.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 120 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
20	313	[1]160 35 607	193 52 985	218 71 1,442	233 91 1,968	241 113 2,550
30	512	153 44 822	179 61 1,202	197 79 1,644	209 98 2,140	213 118 2,690
40	725	146 54 1,044	167 70 1,422	180 87 1,850	188 105 2,328	190 124 2,863
50	950	141 64 1,277	156 79 1,653	166 95 2,076	170 113 2,539	169 131 3,045
60	1,185	136 74 1,523	147 89 1,900	153 105 2,323	154 121 2,780	151 138 3,263
70	1,428	132 84 1,782	139 99 2,169	141 115 2,599	139 131 3,049	134 147 3,505
80	1,679	128 94 2,046	132 109 2,436	131 124 2,863	127 140 3,298	120 145 3,440
90	1,937	125 104 2,317	125 119 2,711	122 134 3,134	117 139 3,282	110 142 3,366
100	2,200	122 114 2,592	120 129 2,987	115 135 3,172	109 139 3,267	103 140 3,306
110	2,469	119 125 2,872	115 133 3,108	110 137 3,214	104 139 3,267	97 139 3,267
120	2,744	117 132 3,072	112 136 3,194	106 138 3,264	99 139 3,280	93 138 3,246
130	3,023	115 136 3,195	109 139 3,286	103 140 3,321	96 140 3,303	90 138 3,238
140	3,307	113 144 3,408	106 145 3,446	99 144 3,424	91 141 3,350	85 137 3,232

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 10.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 140 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
30	512	[1]174 46 852	201 64 1,270	219 83 1,757	230 104 2,301	233 126 2,898
40	725	168 56 1,079	188 73 1,496	202 91 1,965	209 111 2,486	209 132 3,066
50	950	162 65 1,312	178 82 1,723	187 100 2,190	190 119 2,710	188 139 3,268
60	1,185	157 75 1,555	168 91 1,971	174 109 2,443	174 127 2,949	170 146 3,481
70	1,428	152 85 1,816	160 102 2,244	162 119 2,719	159 137 3,219	153 155 3,729
80	1,679	149 96 2,082	152 112 2,529	151 129 2,997	146 146 3,488	138 158 3,842
90	1,937	145 106 2,356	146 122 2,795	142 139 3,268	135 152 3,667	126 157 3,795
100	2,200	142 116 2,633	140 132 3,075	134 148 3,536	126 153 3,681	117 155 3,749
110	2,469	139 126 2,915	135 142 3,363	127 150 3,603	119 153 3,694	110 154 3,710
120	2,744	137 136 3,202	130 148 3,547	122 152 3,666	113 154 3,710	104 153 3,680
130	3,023	135 146 3,488	127 152 3,660	118 155 3,735	108 155 3,734	99 152 3,661
140	3,307	133 152 3,653	124 156 3,756	114 157 3,802	104 156 3,759	95 152 3,648

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 11.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 160 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
30	512	[1]196 47 881	224 66 1,336	242 87 1,866	251 110 2,459	253 133 3,099
40	725	189 57 1,112	210 76 1,369	223 96 2,084	229 117 2,652	229 139 3,282
50	950	183 67 1,348	199 85 1,803	208 104 2,310	211 125 2,874	207 146 3,480
60	1,185	178 76 1,590	189 94 2,046	195 113 2,560	194 133 3,119	189 154 3,708
70	1,428	173 87 1,849	181 104 2,314	183 123 2,835	179 143 3,384	172 162 3,945
80	1,679	169 97 2,117	173 115 2,604	172 133 3,120	166 152 3,661	156 171 4,207
90	1,937	166 107 2,393	167 125 2,872	163 143 3,388	155 161 3,918	144 170 4,196
100	2,200	163 117 2,672	161 135 3,159	154 153 3,674	144 166 4,063	133 170 4,174
110	2,469	160 128 2,956	155 145 3,454	147 162 3,954	136 167 4,099	124 169 4,146
120	2,744	157 137 3,235	150 155 3,745	140 165 4,043	129 168 4,131	117 168 4,122
130	3,023	155 148 3,533	146 163 3,991	135 168 4,124	123 169 4,158	111 167 4,098
140	3,307	153 158 3,826	142 168 4,116	130 171 4,201	118 170 4,186	106 166 4,080

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 12.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 180 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
40	725	[1]211 58 1,143	232 78 1,639	245 100 2,198	250 122 2,811	249 146 3,480
50	950	204 68 1,383	221 88 1,881	229 108 2,426	231 130 3,034	227 153 3,686
60	1,185	199 78 1,629	211 97 2,125	216 118 2,675	214 139 3,286	208 161 3,916
70	1,428	194 88 1,885	202 107 2,384	204 127 2,947	200 148 3,543	191 169 4,162
80	1,679	190 98 2,151	194 117 2,672	193 138 3,239	186 158 3,830	175 178 4,428
90	1,937	186 108 2,428	187 127 2,949	183 147 3,511	174 167 4,099	162 183 4,577
100	2,200	183 119 2,10	181 138 3,239	174 157 3,805	164 176 4,372	150 183 4,573
110	2,469	180 129 2,995	176 148 3,538	167 167 4,099	154 180 4,470	140 183 4,561
120	2,744	178 139 3,275	171 158 3,833	160 177 4,382	146 181 4,525	132 182 4,532
130	3,023	175 149 3,576	166 169 4,142	153 180 4,485	139 183 4,567	125 182 4,550
140	3,307	173 159 3,871	162 178 4,429	148 183 4,575	133 184 4,601	119 181 4,513

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 13.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 200 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
50	950	[1]226 69 1,416	242 91 1,955	251 113 2,552	252 136 3,193	246 160 3,899
60	1,185	220 79 1,665	232 100 2,207	237 122 2,795	235 145 3,448	227 168 4,135
70	1,428	215 89 1,921	223 110 2,462	224 131 3,062	220 154 3,709	210 176 4,375
80	1,679	211 99 2,183	215 120 2,735	213 141 3,333	206 163 3,973	195 184 4,612
90	1,937	207 110 2,462	208 130 3,026	203 152 3,640	194 173 4,279	180 194 4,901
100	2,200	204 120 2,745	202 140 3,316	195 161 3,929	183 182 4,548	168 195 4,956
110	2,469	201 130 3,033	196 151 3,619	187 171 4,228	173 191 4,825	157 196 4,963
120	2,744	198 140 3,318	191 161 3,917	179 182 4,532	164 194 4,896	148 196 4,963
130	3,023	195 151 3,617	186 172 4,231	172 191 4,824	156 196 4,958	140 195 4,956
140	3,307	193 161 3,913	182 182 4,530	166 195 4,928	150 197 5,002	133 195 4,939

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 14.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 220 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
60	1,185	[1]241 81 1,700	253 103 2,283	258 126 2,920	255 150 3,604	246 175 4,340
70	1,428	236 91 1,957	244 113 2,543	245 135 3,177	240 159 3,868	229 183 4,581
80	1,679	232 101 2,225	236 123 2,819	234 145 3,462	226 169 4,148	213 192 4,840
90	1,937	228 111 2,495	229 133 3,098	224 155 3,751	214 178 4,423	199 200 5,107
100	2,200	224 121 2,780	222 143 3,392	215 166 4,051	203 188 4,31	186 208 5,341
110	2,469	221 132 3,069	216 153 3,697	207 176 4,356	192 198 5,023	174 208 5,355
120	2,744	218 142 3,356	211 164 4,002	199 186 4,671	183 205 5,262	164 209 5,369
130	3,023	216 152 3,656	206 174 4,316	192 196 4,980	174 208 5,330	156 209 5,365
140	3,307	213 162 3,954	202 184 4,618	186 205 5,262	167 209 5,387	148 208 5,355

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

Table 15.—*Simulated elm-ash-cottonwood stand yield from an initial condition of 240 trees per acre*

Initial basal area	Initial volume	Elapsed years from initial conditions				
		5	10	15	20	25
60	1,185	[1]262 82 1,733	275 106 2,357	279 130 3,033	276 156 3,68	266 182 4,551
70	1,428	257 92 1,993	265 115 2,620	266 140 3,295	260 165 4,032	248 190 4,791
80	1,679	253 102 2,259	257 125 2,889	255 149 3,571	246 174 4,304	233 199 5,055
90	1,937	248 112 2,528	250 135 3,176	245 160 3,873	234 184 4,612	218 208 5,344
100	2,200	245 122 2,813	243 146 3,468	235 170 4,175	222 194 4,900	205 217 5,614
110	2,469	242 133 3,104	237 156 3,773	227 180 4,478	212 203 5,197	192 221 5,735
120	2,744	239 143 3,392	232 167 4,081	219 190 4,797	202 213 5,510	181 221 5,756
130	3,023	236 153 3,693	227 177 4,393	212 201 5,110	193 219 5,690	172 221 5,765
140	3,307	234 163 3,989	222 187 4,701	205 210 5,414	185 221 5,757	164 221 5,760

[1]The first number of each trio is the simulated yield in number of trees per acre, the second is the simulated yield in basal area (sq ft/acre), and the third is simulated volume yield (cu ft/acre).

APPENDIX II

FORTRAN PROGRAM FOR ELM-ASH-COTTONWOOD YIELD SIMULATION

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C*********************************************************************
C  THIS IS A SIMPLE PROGRAM THAT INTEGRATES THE SYSTEM OF
C  ELM-ASH-COTTONWOOD DIFFERENTIAL EQUATIONS DESCRIBING
C  CHANGE IN NUMBER OF TREES AND BASAL AREA PER ACRE.
C  THIS PROGRAM CALLS THE IBM SCIENTIFIC SUBROUTINE RKGS
C  WHICH DOES THE ACTUAL NUMERICAL INTEGRATION.  FOR MORE
C  INFORMATION ON RKGS REFER TO :
C       INTERNATIONAL BUSINESS MACHINES CORPORATION. 1968.
C       SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE
C       (360-CM-03X) VERSION III PROGRAMMER'S MANUAL.
C       4TH ED. TECHNICAL PUBLICATIONS DEPARTMENT, WHITE PLAINS,
C       NEW YORK.  452 P.
C  INPUTS REQUIRED ARE INITIAL NUMBER OF TREES PER ACRE,
C  INITIAL BASAL AREA PER ACRE, LENGTH OF DESIRED PROJECTION,
C  AND FREQUENCY OF PRINTED OUTPUT.  ALL VALUES ARE FOR TREES
C  5 INCHES DBH AND LARGER.
C*********************************************************************
      EXTERNAL FCT,OUTP,RKGS
      DIMENSION PRMT(5),Y(7),DERY(7),AUX(8,7)
      COMMON TNT1,BAT1,TIMER,PERIOD,REPORT
      TIMER = 0.0

C*********************************************************************
C  READ IN INITIAL NUMBER OF TREES, INITIAL BASAL AREA,
C  PROJECTION PERIOD, AND REPORTING INTERVAL.
C       TNT1 = NUMBER OF TREES PER ACRE 5 INCHES DBH OR LARGER
C       BAT1 = BASAL AREA OF THE TNT1 TREES
C       PERIOD = DESIRED PROJECTION LENGTH IN YEARS
C       REPORT = YEARS BETWEEN REPORTED OUTPUT
C*********************************************************************
      READ(5,500)TNT1,BAT1,PERIOD,REPORT
  500 FORMAT(2F4.0,2F3.0)

C*********************************************************************
C  INITIALIZE ARRAY PRMT (USED BY RKGS)
C       PRMT(1) = LOWER BOUND OF INTEGRATION
C       PRMT(2) = UPPER BOUND OF INTEGRATION
C       PRMT(3) = INITIAL STEP SIZE
C       PRMT(4) = ERROR TOLERANCE
C*********************************************************************
      PRMT(1) = 0.0
      PRMT(2) = PERIOD + .1
      PRMT(3) = 0.1
      PRMT(4) = 0.001
```

```
C*****************************************************************
C  INITIALIZE ARRAY Y WITH THE INITIAL CONDITIONS FOR EACH
C  OF THE DIFFERENTIAL EQUATIONS IN THE SIMULATOR.
C       Y(1) = CUMULATIVE NUMBER OF INGROWTH TREES PER ACRE
C       Y(2) = CUMULATIVE NUMBER OF MORTALITY TREES PER ACRE
C       Y(3) = NUMBER OF TREES PER ACRE.
C       Y(4) = CUMULATIVE BASAL AREA INGROWTH PER ACRE
C       Y(5) = CUMULATIVE BASAL AREA SURVIVOR GROWTH PER ACRE
C       Y(6) = CUMULATIVE BASAL AREA MORTALITY PER ACRE
C       Y(7) = BASAL AREA PER ACRE
C  VALUES MUST BE GREATER THAN ZERO.
C*****************************************************************
      Y(1) = 0.00001
      Y(2) = 0.00001
      Y(3) = TNT1
      Y(4) = 0.00001
      Y(5) = 0.00001
      Y(6) = 0.00001
      Y(7) = BAT1

C*****************************************************************
C  INITIALIZE THE ARRAY NDIM TO THE TOTAL NUMBER OF EQUATIONS
C*****************************************************************
      NDIM=7

C*****************************************************************
C  INITIALIZE THE ARRAY OF EQUATION WEIGHTS, DERY.
C*****************************************************************
      DO 10 I=1,NDIM
      DERY(I)=1.0/FLOAT(NDIM)
   10 CONTINUE


C*****************************************************************
C  WRITE OUTPUT TABLE HEADING.  CFV IS CUBIC FOOT VOLUME
C  ESTIMATED AS A FUNCTION OF BASAL AREA.
C*****************************************************************
      CFV=8.319*Y(7)**1.2112
      WRITE(6,600)TNT1,BAT1,CFV
  600 FORMAT(1H1,17X,34HSIMULATED ELM-ASH-COTTONWOOD YIELD,/
     +10X,51H(PER ACRE VALUES FOR TREES 5 IN. D.B.H. AND LARGER),/19X,
     +31H(TREES 5 INCHES DBH AND LARGER)//19X,
     +23HINITIAL NUMBER OF TREES,
     +F6.0/19X,27HINITIAL BASAL AREA (FT.SQ.),F6.0/
     +19X,25HINITIAL CUBIC FOOT VOLUME,F6.0///
     +60H        ELAPSED          NUMBER          BASAL      CUBIC FOOT/
     +60H           TIME       OF TREES           AREA          VOLUME/)

C*****************************************************************
C  CALL THE RKGS SUBROUTINE TO INTEGRATE THE SYSTEM OF
C  DIFFERENTIAL EQUATIONS.
C*****************************************************************
      CALL RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
      STOP
      END
```

```
      SUBROUTINE FCT(X,Y,DERY)
      DIMENSION Y(7),DERY(7)
      COMMON TNT1,BAT1,TIMER,PERIOD,REPORT
C*********************************************************************
C  THIS SUBROUTINE HOLDS THE RIGHT HAND SIDES OF THE
C  DIFFERENTIAL EQUATIONS TO BE SOLVED SIMULTANEOUSLY.
C  DERY(1) IS THE DERIVATIVE OF Y(1), DERY(2) IS THE
C  DERIVATIVE OF Y(2), ETC.
C*********************************************************************
      DERY(1) = 10.139 * EXP(0.0024561 * Y(3) - 0.015602 * Y(7))

      DERY(2) = 1.1061 * EXP(0.0065612 * Y(7))

      DERY(3) = DERY(1) - DERY(2)

      DERY(4) = 0.13635 * DERY(1)

      DERY(5) = 0.030429 * SQRT(Y(7) * Y(3))

      DERY(6) = (((3.1424*((-ALOG(1.0-RANF(0)))**1.1041)) + 5.0)
     2**2)*0.005454*DERY(2)
      QUAD = Y(7)/Y(3)
      IF(QUAD.GT.1.1.OR.QUAD.LT.0.40) DERY(6) = QUAD*DERY(2)

      DERY(7) = DERY(4) + DERY(5) - DERY(6)

      RETURN
      END



      SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
      DIMENSION Y(NDIM),DERY(NDIM),PRMT(5)
      COMMON TNT1,BAT1,TIMER,PERIOD,REPORT

C*********************************************************************
C  SUBROUTINE OUTP PRINTS SELECTED OUTPUT FROM THE SOLUTION
C  PROCESS.  THIS SUBROUTINE IS CALLED AT LEAST ONCE EVERY
C  PRMT(3) YEARS.  AT EACH CALL TO OUTPUT THE VALUES OF
C  ARRAY Y (THE CURRENT CUMULATIVE YIELD VALUES) AND X
C  (THE TIME VARIABLE) ARE PASSED TO SUBROUTINE OUTP.  THESE
C  VALUES ARE PRINTED EVERY "REPORT" YEARS.
C*********************************************************************
      ELAPSED=X-TIMER
      IF(ELAPSED.GE.(REPORT-.001))GO TO 5
      GO TO 10

    5 CFV=8.319*Y(7)**1.2112
      WRITE(6,600)X,Y(3),Y(7),CFV
  600 FORMAT(4(5X,F10.0))
     +,Y(6)
      TIMER = TIMER + REPORT

   10 RETURN
      END
```

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Shifley, Stephen R.; Moser, John W., Jr.; Brown, Kenneth M.
Growth and yield model for the elm-ash-cottonwood type in Indiana. Res. Pap. NC-218. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station: 1982. 16 p.

Presents a growth and yield model for the elm-ash-cottonwood forest type in Indiana and discusses model validity. Gives simulated yield estimates for a wide range of stand conditions.

KEY WORDS: Bottomland hardwoods, yield tables, simulation.